

Exact Results for a Kondo Problem in One Dimensional t - J Model

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We propose an integrable Kondo problem in a one-dimensional (1D) $t - J$ model. With the open boundary condition of the wave functions at the impurity sites, the model can be exactly solved via Bethe ansatz for a class of $J_{R,L}$ (Kondo coupling constants) and $V_{L,R}$ (impurity potentials) parametrized by a single parameter c . The integrable value of $J_{L,R}$ runs from negative infinity to positive infinity, which allows us to study both the ferromagnetic Kondo problem and antiferromagnetic Kondo problem in a strongly correlated electron system. Generally, there is a residual entropy for the ground state, which indicates a typical non-Fermi liquid behavior.

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Recently, considerable attention has been drawn by the theory of impurities in both the Fermi and Luttinger liquids, and many new developments have been addressed. This renewed interest in the quantum impurity problems was partially stimulated by the search for non-Fermi-liquid fixed points beyond the well known Luttinger liquid and its plausible relevance to the theory of heavy fermions and cuprate superconductors. A great progress is the study on the multi-channel Kondo problem, which has been extensively studied by the conformal field theory in the presence of boundary [1,2] and Bethe ansatz [3–5]. These powerful methods allow to obtain the low temperature thermodynamics near the critical point. Using simpler bosonization and renormalization group techniques, Kane and Fisher have shown that a potential scatter center embedded in a Luttinger liquid is driven to a strong-coupling fixed point by the repulsive electron-electron interactions [6]. This is the first time to show that a single impurity in a Luttinger liquid behaves rather different from that in a Fermi liquid, and directly stimulates the study on the problem of local perturbations to a Luttinger liquid and especially on the Kondo problem in a Luttinger liquid. The Luttinger-Kondo problem was first considered by Lee and Toner [7], who found the crossover of the Kondo temperature from power law dependence on the Kondo coupling constant to an exponential one. Subsequently, a poor man's scaling was performed by Furusaki and Nagaosa [8], who addressed a conjecture which states that ferromagnetic Kondo screening may occur in a Luttinger liquid. The boundary conformal field theory [9] gave out a classification of critical behavior for a Luttinger liquid coupled to a magnetic impurity (without impurity potential). It turns out that there are only two possibilities, a local Fermi liquid with standard low-temperature thermodynamics or a non-Fermi liquid observed by Furusaki and Nagaosa. It is now clear that the non-Fermi-liquid behavior is induced by the tunneling effect of conduction electrons through the impurity which depends only on the bulk properties but not on the detail of the impurity [10,11].

Despite these important progress, the problem of few impurities (potential, magnetic, especially both) embedded in a strongly correlated system is still not well understood. We remark that there are a few progress related to this problem: A spin $S > 1/2$ impurity in a spin $1/2$ Heisenberg chain solved many years ago by Andrei and Johannesson [12] and generalized to arbitrary spins by Lee and Schlottmann, and Schlottmann [13]; an integrable impurity in the supersymmetric $t - J$ model [14] with a very complicated hamiltonian solved by Bedürftig et al. We remark here that these models are somewhat special for the absence of backward scattering off the impurity, and therefore are hard to give a deep understanding for the Kondo problem in a Luttinger liquid, where the backward scattering is crucial to the fixed point of the system. In addition, Wang and Voit has proposed an integrable model of a single magnetic impurity in a δ -potential Fermi gas [11] with a special value of Kondo coupling constant.

Attempting to understand effects of magnetic impurities in a strongly correlated electron system, we study the properties of the integrable $t - J$

model coupled to two magnetic impurities sited at the ends of the system. Our starting point is the following hamiltonian

$$\begin{aligned}
 H &= H_0 + H_i, \\
 H_0 &= - \sum_{j=1, \sigma}^{N_a-1} (C_{j\sigma}^\dagger C_{j+1\sigma} + h.c.) + 2 \sum_{j=1}^{N_a-1} [\mathbf{S}_j \cdot \mathbf{S}_{j+1} + V n_j n_{j+1}], \\
 H_i &= J_L \mathbf{S}_1 \cdot \mathbf{S}_L + V_L n_1 + J_R \mathbf{S}_{N_a} \cdot \mathbf{S}_R + V_R n_{N_a},
 \end{aligned} \tag{1}$$

where $C_{j\sigma}(C_{j\sigma}^\dagger)$ are annihilation (creation) operators of the conduction electrons; V , $J_{R,L}$, $V_{R,L}$ are the nearest neighbor interaction constant, the Kondo coupling constants and the impurity potentials respectively; $\mathbf{S}_j = \frac{1}{2} \sum_{\sigma,\sigma'} C_{j\sigma}^\dagger \sigma_{\sigma\sigma'} C_{j\sigma'}$ is the spin operator of the conduction electrons; $\mathbf{S}_{L,R}$ are the local moments with spin-1/2 sited at the left and right end of the system respectively; $n_j = C_{j\uparrow}^\dagger C_{j\uparrow} + C_{j\downarrow}^\dagger C_{j\downarrow}$ are the number operator of conduction electrons; N_a is the length or site number of the system. Notice that the single occupation condition $n_j \leq 1$ is assumed for the hamiltonian (1). We remark that the model is very resonable for the absence of redundant terms in the hamiltonian.

It is well known that H_0 is exactly solvable for $V = -1/4, 3/4$ [15,16]. In this letter, we only study $V = 3/4$ case while both the charge- and spin-sector can be described by a Luttinger liquid. The $V = -1/4$ (supersymmetric $t - J$ model) can be followed without any difficulty. By including the impurities, any electron impinging on the boundaries will be completely reflected and suffer a reflection matrix $R_{j,L}$ or $R_{j,R}$ [11]. The waves are therefore reflected at either end as

$$\begin{aligned} e^{ik_j x} &\rightarrow R_{j,L}(k_j) e^{-ik_j x}, \quad x \sim 1, \\ e^{ik_j x} &\rightarrow R_{j,R}^{-1}(k_j) e^{-ik_j x - 2ik_j N_a}, \quad x \sim N_a \end{aligned} \quad (2)$$

The reflecting Yang-Baxter equation [17,18]

$$S_{jl}(q_j - q_l) R_{j,a}(q_j) S_{jl}(q_j + q_l) R_{l,a}(q_l) = R_{l,a}(q_l) S_{jl}(q_j + q_l) R_{j,a}(q_j) S_{jl}(q_j - q_l), \quad a = R, L, \quad (3)$$

constrains the integrability of the present model. Here S_{jl} is the electron-electron scattering matrix in the bulk. By evaluating eq.(3), we obtain that the present model is exactly solvable with the following parametrized J_a and V_a ($a = L, R$): $J_a = -8/(2c_a + 3)(2c_a - 1)$, $V_a = (4c_a^2 - 7)/(2c_a + 3)(2c_a - 1)$, where $c_{L,R}$ are two arbitrary real constants. Generally, J_L and J_R may take different values. Here we consider only $J_R = J_L$, $V_R = V_L$, i.e., $c_L = c_R = c$ case. The solution of our model in the integrable line is quite similar to those of other integrable models with open boundaries [17,18]. Notice that the reflection matrix $R_{L,R}$ (K_\pm in ref.[17]) is an operator one rather than a c-number matrix. The spectra of the hamiltonian (1) are uniquely determined by the following Bethe ansatz equations

$$\begin{aligned} \left(\frac{q_j - \frac{i}{2}}{q_j + \frac{i}{2}}\right)^{2N_a} &= \left[\frac{q_j + i(c-1)}{q_j - i(c-1)}\right]^2 \prod_{r=\pm 1} \left\{ \prod_{l \neq j}^N \frac{q_j - r q_l - i}{q_j - r q_l + i} \prod_{\alpha=1}^M \frac{q_j - r \lambda_\alpha + \frac{i}{2}}{q_j - r \lambda_\alpha - \frac{i}{2}} \right\} \\ \left\{ \frac{\lambda_\alpha - i(c - \frac{1}{2})}{\lambda_\alpha + i(c - \frac{1}{2})} \frac{\lambda_\alpha + i(c + \frac{1}{2})}{\lambda_\alpha - i(c + \frac{1}{2})} \right\}^2 &\prod_{r=\pm 1} \prod_{j=1}^N \frac{\lambda_\alpha - r q_j + \frac{i}{2}}{\lambda_\alpha - r q_j - \frac{i}{2}} = \prod_{r=\pm 1} \prod_{\beta \neq \alpha}^M \frac{\lambda_\alpha - r \lambda_\beta + i}{\lambda_\alpha - r \lambda_\beta - i}, \end{aligned} \quad (4)$$

with the eigenvalue of (1) given by $E = 2N - \sum_{j=1}^N 4/(4q_j^2 + 1)$. Here N is the number of conduction electrons, $q_j = 1/2 \tan(k_j/2)$, and k_j and λ_α are the rapidities of charge and spin respectively. M is the number of spin-down electrons.

Below we discuss the ground state properties for different regions of parameter c .

(i) $c \geq 1$. The system falls into the ferromagnetic Kondo coupling regime and no bound state can exist at low energy scales. The ground state is thus described by two sets of real parameters $\{q_j\}$ and $\{\lambda_\alpha\}$. Define the quantities

$$\begin{aligned} Z_{N_a}^c(q) &= \frac{1}{\pi} \left\{ -\theta_1(q) + \frac{1}{2N_a} [\phi_c(q) - \sum_{\alpha=-M}^M \theta_1(q - \lambda_\alpha) + \sum_{j=-N}^N \theta_2(q - q_j)] \right\}, \\ Z_{N_a}^s(\lambda) &= \frac{1}{2N_a} \left\{ \phi_s(\lambda) - \sum_{j=-N}^N \theta_1(\lambda - q_j) + \sum_{\alpha=-M}^M \theta_2(\lambda - \lambda_\alpha) \right\}, \end{aligned} \quad (5)$$

with the phase shifts $\phi_c(q) = -\theta_2(q) + 4 \tan^{-1}[q/(c-1)]$, $\phi_s(\lambda) = -\theta_2(\lambda) + 4 \tan^{-1}[\lambda/(c+1/2)] - 4 \tan^{-1}[\lambda/(c-1/2)]$ induced by the impurity in the charge- and spin-sector respectively, and $\theta_n(x) = -2 \tan^{-1}(2x/n)$. Note above we have used the reflection symmetry of the Bethe ansatz equations to include solutions with $q_{-j} = -q_j$ and $\lambda_{-\alpha} = -\lambda_\alpha$. The Bethe ansatz equations are solved by $Z_{N_a}^c(q_j) = I_j/N_a$ and $Z_{N_a}^s(\lambda) = J_\alpha/N_a$, where I_j and J_α are non-zero integers. In the ground state, I_j and J_α must be consecutive numbers around zero symmetrically to minimize the energy. The roots q_j and λ_α becomes dense in the thermodynamic limit, and we define their densities as

$$\rho_{N_a}^c(q) = \frac{dZ_{N_a}^c(q)}{dq}, \quad \rho_{N_a}^s(\lambda) = \frac{dZ_{N_a}^s(\lambda)}{d\lambda}. \quad (6)$$

The cutoffs of q and λ in the ground state are $\pm Q$ and $\pm\Lambda$ respectively, which correspond to $Z_{N_a}^c(\pm Q) = \pm(N + 1/2)/N_a$, $Z_{N_a}^s(\pm\Lambda) = \pm(M + 1/2)/N_a$. In the thermodynamic limit $N_a \rightarrow \infty$, $N \rightarrow \infty$, and $N/N_a \rightarrow \text{finite}$, we find that the energy is minimized at $\Lambda \rightarrow \infty$, which gives a result of $N = 2M$ by integrating eq.(6). This leaves a spin triplet ground state which is contradicting to the Furusaki-Nagaosa conjecture [8]. Notice that the magnetization is given by $1/2(N + 2 - 2M)$.

(ii) $1/2 < c < 1$. The system falls also into the ferromagnetic Kondo coupling regime. However, unlike case (i), two bound states of electrons can be formed around the impurities in the ground state, which correspond to two imaginary q modes at $q = \pm i(1 - c)$. The Bethe ansatz equations for the real modes are thus reduced to

$$\begin{aligned} \left(\frac{q_j - \frac{i}{2}}{q_j + \frac{i}{2}}\right)^{2N_a} &= \left[\frac{q_j + i(c-1)}{q_j - i(c-1)} \frac{q_j - ic}{q_j + ic} \frac{q_j + i(c-2)}{q_j - i(c-2)}\right]^2 \times \\ &\times \prod_{r=\pm 1} \left\{ \prod_{l \neq j}^{N-2} \frac{q_j - r q_l - i}{q_j - r q_l + i} \prod_{\alpha=1}^M \frac{q_j - r \lambda_\alpha + \frac{i}{2}}{q_j - r \lambda_\alpha - \frac{i}{2}} \right\} \\ &\left\{ \frac{\lambda_\alpha - i(c - \frac{3}{2})}{\lambda_\alpha + i(c - \frac{3}{2})} \frac{\lambda_\alpha + i(c + \frac{1}{2})}{\lambda_\alpha - i(c + \frac{1}{2})} \right\}^2 \prod_{r=\pm 1} \prod_{j=1}^{N-2} \frac{\lambda_\alpha - r q_j + \frac{i}{2}}{\lambda_\alpha - r q_j - \frac{i}{2}} = \prod_{r=\pm 1} \prod_{\beta \neq \alpha}^M \frac{\lambda_\alpha - r \lambda_\beta + i}{\lambda_\alpha - r \lambda_\beta - i}. \end{aligned} \quad (7)$$

Following the same procedure discussed above, we obtain again a spin triplet ground state. This can be understood in the following picture: The bounded electrons and the local moments form two spin-1 local composites due to the effective attraction and the ferromagnetic Kondo coupling. However, the itinerant electrons impinging on these composites will screen their moments partially due to the indirect Kondo coupling induced by the electron-electron correlation. In such a sense, we recover Furusaki-Nagaosa's conjecture [8], though the local moments are not completely screened.

(iii) $-1/2 < c < 1/2$. The system falls into the antiferromagnetic Kondo coupling regime. No bound state appears in the ground state. By integrating eq.(6) in the thermodynamic limit, we have $N + 2 = 2M$. This indicates a spin singlet ground state, a similar result to that of the Kondo problem in a Fermi liquid.

(iv) $-1 \leq c \leq -1/2$. No bound state exists in the ground state. It seems that the ground state should be a spin triplet. We note that both J_a and V_a take positive values in this region. The repulsive boundary potential dominates over the Kondo coupling. It will repel the conduction electrons to form singlet with the local moments and thus make the Kondo coupling ineffective. In such a sense, no Kondo screening occurs, which strongly indicates that both the boundary potential and the electron-electron correlation in the bulk have significant effects to the Kondo problem in a Luttinger liquid.

(v) $-3/2 < c < -1$. For this case, the system is still in the regime of antiferromagnetic Kondo coupling but with a weaker or attractive boundary potential. Two imaginary λ mode at $\lambda = \pm i(c + 1/2)$ and two imaginary q modes at $q = \pm i(c + 1)$ appear in the ground state. These modes correspond to the formation of bound singlet pairs of two conduction electrons with the local moments. By integrating the densities of the real modes in the thermodynamic limit, we still arrive at a spin singlet ground state.

(vi) $c < -3/2$. The Kondo coupling is ferromagnetic and the boundary potential is repulsive. There is no bound state in the ground state. Following the same procedure discussed in (i), we obtain again $N = 2M$ for the ground state. Therefore, there is no Kondo screening in this region, which contradicts to the Furusaki-Nagaosa conjecture.

The thermodynamics of the present model can be calculated in a closed form based on the Bethe ansatz equations (4). This allows us to obtain the temperature and magnetic field dependence of the free energy which contains three parts of contributions, i.e., the bulk term, the boundary term and the Kondo effect term. Here we omit the detail of calculation following the standard procedure which can be found in some excellent works [3,19-21]. The Kondo effect induced free energy is the most interesting one which takes the following form at low temperatures (Hereafter we assume c is an integer or half integer)

$$F_k = -T \text{sign}(n_1) \int \frac{\ln[1 + \zeta_{|n_1|}(\lambda)] d\lambda}{2 \cosh(\pi\lambda)} - T \text{sign}(n_2) \int \frac{\ln[1 + \zeta_{|n_2|}(\lambda)] d\lambda}{2 \cosh(\pi\lambda)}, \quad (8)$$

where n_1 and n_2 are two c -dependent integers, $\zeta_n(\lambda)$ are elements of the following coupled integral equations

$$\begin{aligned} \ln \eta(\lambda) &= \frac{\epsilon_0(\lambda) - \mu}{T} - ([1]G + [2]) \ln[1 + \eta^{-1}(\lambda)] - G \ln[1 + \zeta_1(\lambda)], \\ \ln \zeta_n(\lambda) &= G \{ \ln[1 + \zeta_{n+1}(\lambda)] + \ln[1 + \zeta_{n-1}(\lambda)] \}, \quad n > 1, \end{aligned}$$

$$\ln \zeta_1(\lambda) = -G \ln[1 + \eta^{-1}(\lambda)] + G \ln[1 + \zeta_2(\lambda)], \quad (9)$$

$$\lim_{n \rightarrow \infty} \{[n] \ln[1 + \zeta_{n+1}(\lambda)] - [n+1] \ln[1 + \zeta_n(\lambda)]\} = \frac{2H}{T},$$

where μ is the chemical potential and $\epsilon_0(\lambda) = 2 - 4\lambda^2/(4\lambda^2 + 1)$; $[n]$ and G are integral operators with the kernels $a_n(\lambda) = (n/2\pi)/[\lambda^2 + (n/2)^2]$ and $1/[2 \cosh(\pi\lambda)]$ respectively; H is the external magnetic field. Notice that we have omitted the excitations breaking the bound states in deriving (9) for the energy gaps associated with them. Their contributions to the free energy are exponentially small at low enough temperatures.

For $H = 0$ and $T \rightarrow 0$, the driving term $G \ln[1 + \eta^{-1}(\lambda)]$ in (9) is nothing but $T^{-1}\epsilon_s(\lambda)$ with $\epsilon_s(\lambda)$ the dressed energy of the spin waves which has the asymptotic form $\epsilon_s(\lambda) = 2\pi e_0 \exp(-\pi|\lambda|)$ for $|\lambda| \rightarrow \infty$, where e_0 is the energy density of the ground state. This allows us to formulate the low-temperature expansion of (9). The asymptotic solution of (9) is given by functions $\zeta_n(x)$, $x = \ln[(\pi e_0)/T] - \pi|\lambda|$ which are monotonically decreasing in x for all n and tending to finite limits ζ_{n+} as $x \rightarrow \infty$. The limits are given by $\zeta_{n+} = \sinh^2(nH/T)/\sinh^2(H/T) - 1$. No anomaly appears in the free energy of the bulk and the open boundary. They contain only constant terms and T^2 terms up to the order $O(T^2)$. An interesting feature is that with different c value, the system may show both Fermi- and non-Fermi-liquid behavior. To show this, we study the entropy of the ground state for a variety of c regions. For $c > 1$ or $c < -3/2$, there is no bound state in the ground state. n_1 and n_2 take the values $2|c| + 1$ and $1 - 2|c|$ respectively. This leaves a residual entropy of the ground state as $S_g = \ln[(2|c| + 1)/(2|c| - 1)]$. For $c = 0$, the local moments are completely screened. Both n_1 and n_2 are equal to unit. Thus the entropy of the ground state is zero and the system flows to a Fermi-liquid fixed point. For $c = -1/2$, $n_1 = 2$ and $n_2 = 0$. The residual entropy takes a value of $\ln 2$. While for $c = -1$, $n_1 = -1/2$, $n_2 = 3/2$, $S_g = \ln 3$. Notice that for $c = -1/2$ and $c = -1$, the system falls into the regime of antiferromagnetic Kondo coupling but with a non-zero residual entropy. We remark that $c = 1/2$ and $c = -3/2$ are two critical points since at these points, both J_a and V_a are divergent. The residual entropy has different limits for $c \rightarrow 1/2 + 0^+ (-3/2 + 0^+)$ and $c \rightarrow 1/2 + 0^- (-3/2 + 0^-)$. From the above discussion we conclude that the system generally has a c -dependent residual entropy which strongly indicates a non-Fermi-liquid behavior. The system can flow to a Fermi-liquid fixed point only in a narrow parameter region of $c \sim 0$. Such a fascinating effect can be understood in the following picture: The charge-spin coupling induced by the backward scattering off the impurity introduces an effective boundary field to the local moment, which means the charge degrees of freedom join the Kondo effect. However, unlike a real magnetic field, the “effective field” does not split the degeneracy of different orientation of the local moment. In fact, when the conduction electrons impinging and leaving the impurity, the incident waves and the reflection waves feel different strengths of the impurity spin. One is $1/2 - c$ and the other is $1/2 + c$ or vice versa. The finite residual entropy is a result of cooperative effect of the charge- and spin sectors.

In summary, we introduce an integrable model of Kondo problem in a 1D strongly correlated electron system. With different values of the parameter c , the system can show either a Fermi liquid behavior or a non-Fermi liquid behavior beyond that obtained by Furusaki and Nagaosa [8]. A non-singlet ground state in an antiferromagnetic Kondo coupling region is obtained. It is found that the residual entropy depends not only on the self magnetization of the ground state, but also on the interaction parameter c , which we interpret as a “cooperative effect” of the Kondo coupling and the impurity scattering. It is instructive to apply renormalization group analysis and conformal field theory to reveal a full picture for such an interesting problem.

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